

Non-Chern-Simons Topological Mass Generation in (2+1) Dimensions

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Abstract

By dimensional reduction of a massive B \wedge F theory, a new topological field theory is constructed in (2+1) dimensions. Two different topological terms, one involving a scalar and a Kalb-Ramond fields and another one equivalent to the four-dimensional B \wedge F term, are present. We constructed two actions with these topological terms and show that a topological mass generation mechanism can be implemented. Using the non-Chern-Simons topological term, an action is proposed leading to a classical duality relation between Klein-Gordon and Maxwell actions. We also have shown that an action in (2+1) dimensions with the Kalb-Ramond field is related by Buscher's duality transformation to a massive gauge-invariant Stückelberg-type theory.

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I. INTRODUCTION

It is well known that a topological Chern-Simons term give rise to gauge invariant mass to the gauge field [1]. In (3+1) dimensions, two procedures are generally used for generating massive gauge fields consistent with gauge-invariance. One is the Stückelberg formulation [2] which is the more familiar Higgs mechanism in its simplest form and the other one uses a 2-form potential (Kalb-Ramond field) B coupled to the one-form gauge potential A through a $B \wedge F$ term, where $F = dA$ is the field-strength of A [3–5].

This theory has two types of gauge-invariance and has therefore highly constrained couplings and is very geometrical. In a first order formulation of the non-Abelian Yang-Mills gauge theory (BF-YM model) [6] a $B \wedge F$ term has been used to contribute for a discussion of quark confinement in continuum QCD. Additionally, transmutation of statistics of point particles in (2+1) dimensions can be generalized to that of strings in (3+1) dimensions via a $B \wedge F$ term [7]. On the other hand, the so-called mixed Chern-Simons term, which involves two one-form gauge fields, was recently studied in connection with certain condensed matter systems [8], namely *Josephson junction arrays* [9].

In this letter, we consider a new topological term in (2+1) dimensions, obtained by dimensional reduction from a $B \wedge F$ (3+1) dimensional Abelian model (to the best of our knowledge, this term has not been studied in the explicit form presented here). This term has the form $B \wedge d\phi$ and involves B and a 0-form field ϕ . We show that this term can generate mass for the Kalb-Ramond field as well as for the scalar field. An action with this non-Chern-Simons topological term, leads us to a classical duality between the free Klein-Gordon and Maxwell actions. On the other hand, using the Stückelberg formulation, an alternative massive gauge-invariant model is constructed. Finally, motivated by the fact that a interchange between topological and Noether current usually denotes a duality transformation, we also shown that a topological action in 2+1 dimensions with the Kalb-Ramond field is related by Buscher’s duality transformation [10] to a Stückelberg-type theory.

II. NON-CHERN-SIMONS TOPOLOGICAL TERM

Our starting point is the Abelian $B \wedge F$ four-dimensional action [11,12]

$$S_{BF} = \int_{M_4} \{B \wedge F - g^2 B \wedge *B\}. \quad (1)$$

This action is formulated in terms of the two-form potential B while $F = dA$ is the field-strength of a one-form gauge potential A and $*$ is the Hodge star (duality) operator. The quadratic term is included for latter convenience.

Dimensional reduction is usually done by expanding the fields in normal modes corresponding to the compactified extra dimensions, and integrating out the extra dimensions. This approach is very useful in dual models and superstrings [13]. Here, however, we only consider the fields in higher dimensions to be independent of the extra dimensions.

In this case, we assume that our fields are independent of the extra coordinate x_3 . From (1), on performing dimensional reduction as described above, we get in three dimensions

$$S = \int_{M_3} \{B \wedge d\phi + V \wedge F - g^2 B \wedge *B + g^2 V \wedge *V\}, \quad (2)$$

where V and ϕ are a 1-form and a 0-form fields respectively.

We recognize that $B \wedge d\phi$ is topological in the sense that there is no explicit dependence on the space-time metric. One has to stress that this term may not be confused with the two-dimensional version of the $B \wedge F$, which involves a scalar and a one-form fields [14]. Moreover, a term that is equivalent to the four-dimensional $B \wedge F$ term is present in action (2) (the so-called mixed Chern-Simons term, $V \wedge F$). On the other hand, this action displays a local tensor gauge symmetry whose origin is to be connected to the topological character of model and present still an invariance under $U(1) \times U(1)$.

III. TOPOLOGICAL MASS GENERATION

Now, in order to show the topological mass generation for the vector and tensor fields, we construct two variations from the model (2), by introducing their propagation terms.

The model with propagation for the two-form gauge potential B and with the topological term $B \wedge d\phi$ may be represented through the action

$$S = \int_{M_3} \left\{ \frac{1}{2} H \wedge {}^*H + \frac{1}{2} d\phi \wedge {}^*d\phi + \kappa B \wedge d\phi \right\}, \quad (3)$$

where the second term is a Klein-Gordon term, κ is a mass parameter and $H = dB$ is a three-form field-strength of B .

We follow here the same steps that has been used by Allen *et al.* [5] in order to show the topological mass generation in the context of $B \wedge F$ model. Thus, we find the equations of motion for scalar and tensor fields, which are respectively

$$d {}^*H = \kappa d\phi \quad (4)$$

and

$$d {}^*d\phi = -\kappa H. \quad (5)$$

Applying $d {}^*$ on both sides of eq. (5) and using the eq. (4), we obtain the equation of motion for ϕ , namely

$$(d {}^*d {}^* + \kappa^2) d\phi = 0. \quad (6)$$

Repeating the procedure above in reverse order, we obtain the equation of motion for H

$$(d {}^*d {}^* + \kappa^2) H = 0. \quad (7)$$

These equations can be rewritten as

$$(\square + \kappa^2)\partial_\mu\phi = 0 \quad (8)$$

and

$$(\square + \kappa^2)H = 0. \quad (9)$$

Therefore, the fluctuations of ϕ and H are massive. Obviously, these two possibilities can not occurs simultaneously. Indeed, in the most interesting case, the degree of freedom of the massless ϕ field is "eaten up" by the gauge field B to become massive and the ϕ field completely decouples from the theory.

On the other hand, the model with propagation for two one-form gauge fields is represented by the action

$$S = \int_{M_3} \left\{ \frac{1}{2} F \wedge *F + \kappa V \wedge F + \frac{1}{2} G \wedge *G \right\}, \quad (10)$$

where the first term is the Maxwell one, and

$$G = dV. \quad (11)$$

In the equation (10) we highlight the topological term that involves two vector fields. The equations of motion for them are

$$d^*F = -\kappa dV \quad (12)$$

and

$$d^*G = \kappa F. \quad (13)$$

Following the former procedure we get

$$(\square + \kappa^2)F = 0, \quad (14)$$

and

$$(\square + \kappa^2)G = 0, \quad (15)$$

which shows that the fluctuations of F and G are massive. This last case has already been discussed by Ghosh *et al.* [15].

IV. A CLASSICAL DUALITY

Let us take a look in the following action

$$\int_{M_3} \left[B \wedge d\phi - \frac{1}{2} B \wedge {}^*B \right] \quad (16)$$

We would like to mention the analogy of this action with the so called BF-Yang-Mills model [6]. The latter formulation take advantage of the two-form gauge field B to use a first order formalism to study pure Yang-Mills theory in four dimensions. Furthermore, the BF-YM model (using Stückelberg auxiliary fields) preserves all the symmetries of the topological $B \wedge F$ model, and so, Yang-Mills theory can be viewed as a perturbative expansion around the pure topological theory [12]. In the present case, the Kalb-Ramond field can be also seen as an auxiliary field, leading us to a free Maxwell action. As a matter of fact, the non-Abelian version of (16) may be interesting if we want to treat 3D Yang-Mills theory in a first order formalism.

On the other hand, we can consider the action (16) as a master equation for Klein-Gordon and Maxwell action in (2+1) dimensions. Indeed, is easy to see that variation with respect to ϕ implies that

$$dB = 0$$

and, from the Poincaré's lemma

$$B = dA.$$

Putting this result in (16), we have

$$S_M = -\frac{1}{2} \int_{M_3} F \wedge {}^*F, \quad (17)$$

which is the Maxwell action. Performing now the variation with respect to B , we can write down

$$B = {}^*d\phi.$$

Substituting in (16) gives

$$S_\phi = \frac{1}{2} \int_{M_3} d\phi \wedge {}^*d\phi. \quad (18)$$

So, from the master action (16) we have shown that S_M and S_ϕ are dual to each other [16]. This duality, specially if considered in the framework of non-Abelian extensions, can aids greatly in unraveling interesting features of the models with non-trivial topology. In particular, may be interesting the study toward connection with new gauge formulations of three-dimensional gravity [18]. Further discussions about the consequences of extensions of (16) will be presented in Ref. [17].

V. STÜCKELBERG-TYPE MASS GENERATION

We would like now to introduce a different type of mass generation. The Stückelberg formulation [2] enforces a gauge invariance by means of an auxiliary field. The starting point here is the gauge invariant action:

$$S = \int_{M_3} \left\{ \frac{1}{2} H \wedge *H - m^2 (B - d\Gamma) \wedge *(B - d\Gamma) \right\}. \quad (19)$$

This action has invariance under

$$B \rightarrow B + d\Omega \quad (20)$$

and

$$\Gamma \rightarrow \Gamma + \Omega, \quad (21)$$

where Γ is the Stückelberg one-form auxiliary field and Ω its respective transformation parameter. The equations of motion following from this action for B and Γ are

$$d*H - m^2 *(B - d\Gamma) = 0 \quad (22)$$

and

$$d*(B - d\Gamma) = 0. \quad (23)$$

So the current $K = m^2 *(B - d\Gamma)$ associated with B field is conserved due to equation of motion of the Γ field, such as that for Noether current.

Let us now compare two different formulation of spin-one massive theory, namely the actions (19) and (3). The equation of motion from the latter action for Kalb-Ramond field is

$$*d*H = \kappa *d\phi = J_B. \quad (24)$$

Therefore the current J_B is an algebraically conserved current, since $(*d*)(*d\phi) = 0$ (equivalently, $\partial^\mu J_{\mu\nu} = 0$).

This interchange between topological and Noether current in two different formulations of massive spin-one gauge theory usually denotes a duality transformation. As we shall see now, indeed that is the case.

Consider a global symmetry in (3) of the form $\delta\phi = \epsilon$ and $\delta B = 0$. The dual theory is obtained by the procedure of gauging the global symmetry in the model by a one-form gauge field A and constraining it to be zero by means of a closed 2-form Lagrange multiplier

Φ . Thus this gauge field is integrated and the theory is expressed in terms of the multiplier field. This is the well-known Buscher's duality procedure [10].

Having the above procedure in mind the action (3) may be rewritten as

$$S = \int_{M_3} \left\{ \frac{1}{2} H \wedge *H + \frac{1}{2} (d\phi - A) \wedge *(d\phi - A) + \kappa B \wedge (d\phi - A) + \kappa \Phi \wedge A \right\}, \quad (25)$$

where the 2-form field Φ is defined as $\Phi = d\Gamma$.

Deriving the action (25) respect to A , one gets

$$(d\phi - A) = -\kappa(B - \Phi), \quad (26)$$

and integrating out the field A we reobtain the Stückelberg action (19).

Similar results has been discussed by Harikumar and Sivakumar in the context of four-dimensional $B \wedge F$ model [19].

VI. CONCLUSIONS

In summary, it was shown that a topologically massive theory can arise from three-dimensional models containing a 2-form (Kalb-Ramond) and a 0-form fields. Such models are obtained from dimensional reduction of a $B \wedge F$ four-dimensional theory. Our main analysis has focused on the role played by the non-Chern-Simons topological term, namely, that involving a 2-form and a 0-form fields. We showed that this term can generate mass to the Kalb-Ramond field. Further, this term favors a classical duality equivalence between a massless scalar field theory and a Maxwell action.

Finally, we have shown that a 3D topologically massive theory describing spin-one particle, with a topological term different from the usual $B \wedge F$ term, is dually equivalent to a Stückelberg-type spin-one theory.

Acknowledgments

We wish to thank Marcony Silva Cunha for helpful discussions. This work was supported in part by Conselho Nacional de Desenvolvimento Científico e Tecnológico-CNPq and Fundação Coordenação de Aperfeiçoamento de Pessoal de Nível Superior-CAPES. D. M. M. acknowledges the Universidade Estadual do Ceará-UECE for the great subsidy.

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